Center City Public Charter Schools
Character, Excellence, Service

6th Grade Mathematics

Unit #2: Applying Proportional Reasoning to Problem Solve with Rates and Ratios
Pacing: 32 Days

Unit Overview

This unit introduces students to the concept of a ratio and engages them in using ratio and rate reasoning to solve problems. It builds upon students' work in earlier grades with whole number multiplication and division. The multiplicative reasoning required for working with proportional relationships cuts across most Grade 6 mathematical content domains and is therefore a crucial starting point for the course.

Students represent rate relationships with tape diagrams, tables, graphs, and equations. Students also are introduced to percent of a quantity as a rate per 100. Rate reasoning is central to the study of algebra and is a critical foundation for understanding slope, which is formalized in grades 7 and 8. As students work with unit rates and interpret percent as a rate per 100, and as they analyze the relationships among the values, they look for and make use of structure (MP.7). As students become more sophisticated in their application of ratio reasoning, they learn when it is best to solve problems with ratios, their associated unit rates, or percents (MP.5).

Prerequisite Skills

1) Simplify fractions
2) Identify and create equivalent fractions
3) Able to cross-multiply
4) Identify common units of measure in both the metric and customary systems
5) Relate units of measure to common benchmarks (i.e. a yard is about the length of a baseball bat, etc.)
6) Understand that a unit of measure within any system can be reported in terms of another unit (i.e. 12 inches = 1 foot)
7) Know common conversions within the metric and customary systems

Vocabulary

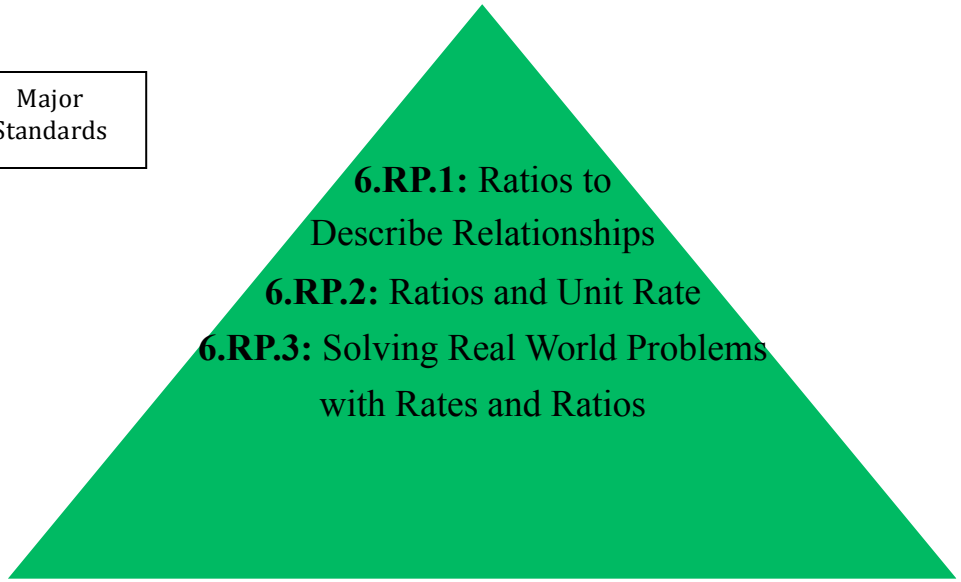
Ratio
Equivalent Ratio
Proportional
Proportionate
Numerator
Denominator
Simplify
Ratio
Comparison
Rate
Unit Rate
Constant speed
Unit Conversion
Convert
Unit of Measure
Capacity
Mass
Metric System
Customary System
Plot
Pairs
Coordinates
Coordinate pair
Coordinate Plan
Equivalent fractions
Ordered pair
Plot
Quantity
Table
X-axis
Y-axis

Mathematical Practices

MP.1: Make sense of problems and persevere in solving them
MP.2: Reason abstractly and quantitatively
MP.3: Construct viable arguments and critique the reasoning of others
MP.4: Model with mathematics
MP.5: Use appropriate tools strategically
MP.6: Attend to precision
MP.7: Look for and make use of structure
MP.8: Look for and express regularity in repeated reasoning

Common Core State Standards

Major Standards



According to the PARCC Model Content Framework,
The key advances in ratio concepts between sixth and seventh grade are:

"Students grow in their ability to analyze proportional relationships. They decide whether two quantities are in a proportional relationship (7.RP.2a); they work with percents, including simple interest, percent increase and decrease, tax, markups and markdowns, gratuities and commission, and percent error (7.RP.3); they analyze proportional relationships and solve problems involving unit rates associated with ratios of fractions (7.RP.1)…"

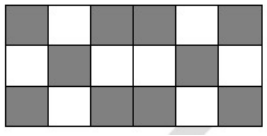
Progression of Skills

5th Grade	6th Grade	7th Grade
N/A	**6.RP.1:** Understand the **concept of a ratio** and use ratio language to **describe a ratio relationship** between two quantities.	**7.RP.2:** Recognize and represent **proportional relationships** between quantities.
N/A	**6.RP.2:** Understand the concept of a **unit rate** a/b associated with a ratio a:b with b ≠ 0, and **use rate language in the context of a ratio relationship**.	**7.RP.1: Compute unit rates** associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
N/A	**6.RP.3:** Use **ratio and rate reasoning to solve real-world and mathematical** problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.	**7.RP.3:** Use proportional relationships to solve **multistep ratio and percent problems.** Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error

Big Ideas	Students Will…	
	Know/Understand	**Be Skilled At…**
• How does a ratio describe the relationship between two quantities? Why is it important to understand the comparison of two numbers as a ratio? • How can unit rates be used to describe relationships between quantities? How would knowing a unit rate be helpful? • What are equivalent ratios and how can they be used? When would you use a table to list equivalent ratios rather than ratios in lowest form? • What is the relationship between ratios and percentages? How is it useful to look at quantity as a percent? In what ways can percent problems be solved? • How does knowing equivalent measurements make it possible to convert measurement units within a system? What is a strategy you could use to convert the measurements? • How can proportions be used to make comparisons and make predictions?	• The concept of a ratio and use ratio language to describe a ratio relationship between two quantities. (6.RP.1) • The concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship (6.RP.2) • The structure of the percent proportion (6.RP.3) • The definition of a rate (6.RP.2) • That a proportion is a statement of equality between two ratios. (6.RP.3.1) • That a proportion represents a relationship between two quantities and may be expressed as a ratio. • That a ratio can be part-to-whole or part-to-part. • The vocabulary used to describe ratios and proportional reasoning. • That a ratio is the comparison of two quantities or measures. • Unit rate as a proportion, using additive and multiplicative reasoning. • The vocabulary used to describe unit rate in the context of a ratio relationship. • multiplicative operations, especially the identity property, and their connection to solving unit rate and ratio problems. • that different tools express the same quantity ratio (e.g. tape diagrams and double number lines show rate reasoning given the same context).	• Finding a percent of a quantity as a rate per 100 solve problems involving finding the whole, given a part and the percent. (6.RP.3) • Using ratio reasoning to convert measurement units; manipulating and transforming units appropriately when multiplying or dividing quantities. (6.RP.3) • Solving unit rate problems including those involving unit pricing and constant speed (6.RP.3.3) • Making tables of equivalent ratios relating quantities with whole-number measurements, finding missing values in the tables, and plotting the pairs of values on the coordinate plane. Using tables to compare ratios (6.RP.3) • Using ratio and rate reasoning to solve real- world and mathematical problems • Making and interpreting tables of equivalent ratios • Using multiple representations such as tape diagrams, double number line diagrams, or equations to solve rate and ratio problems • Solving unit rate problems (Including unit pricing and constant speed) • Creating a unit rate based on information about a ratio relationship. • Solving unit rate problems using multiplicative, additive, and proportional reasoning.

Unit Sequence				
	Student Friendly Objective SWBAT…	**Key Points/ Teaching Tips**	**Sample Assessment Item from Exit Ticket**	**Instructional Resources**
1	Make sense of real world problems by modeling the relationship between two quantities	• Inquiry-based/exploratory lesson, beginning with the "Candies" task – students should not be expected to use ratio notation or language when completing this task, but to use visuals and models to represent, make sense of and solve the problems in the task • Allow time for students to judge the reasonableness of other students' work; push students to explain how they modeled/reasoned about this task • After the class completes the task successfully, introduce them to the concept of "ratio" by illustrating the concept in the task they just completed • define ratio as the comparison of two quantities or measures and challenge them to describe the ratios in this task (i.e. ratio of cups of cream to chocolate for the last problem) • then complete the inquiry lab (again, emphasize visuals and modeling so that students are representing each relationship)	Write a scenario below to show the ratio 2:7.	"Candies" (Appendix C) My Math Chapter 1 Inquiry Lab (Pages 15 – 18)

2 **3**	Compare two quantities in a real world scenario using ratio language and record them in writing using three different ratio notations (1:2, 1/2 and 1 to 2). Attend to precision when using ratios by ensuring the order accurately represents the relationship.	• Have students practice interpreting scenarios that involve ratios by first describing the relationship shown by the ratio and then drawing a diagram to demonstrate meaning. Ask students to describe how the scenario would change if the ratio was reversed. • Work with your students to review how ratio language defines how to set up a ratio between two quantities by setting up a clearly labeled diagram. • Emphasize the importance of order when writing/representing ratios	1) A restaurant worker used 5 loaves of wheat bread and 2 loaves of rye bread to make sandwiches for an event. Which of these ratios compares the number of loaves of rye bread to the number of loaves of wheat bread? A. 2:7 B. 2:5 C. 5:2 D. 5:7 2) In the park there were: • 12 boys playing soccer • 5 girls playing jumped rope • 3 boys playing basketball • 5 girls playing on the slides What is the ratio of number of boys to girls? Write the ratio 3 different ways: 3) An artist is using red, blue, and green tiles to create a mosaic. • The ratio of red tiles to total tiles should be 2:5. • For every 2 blue tiles, there should be 1 green tile. Draw a set of tiles the artist could use. Show or explain how you found your answer.	My Math Chapter 1 Lesson 2 Engage NY Lesson 1 (Appendix C) https://learnzillion.com/lessons/603 https://learnzillion.com/lessons/602-understand-the-difference-between-fractions-and-ratios

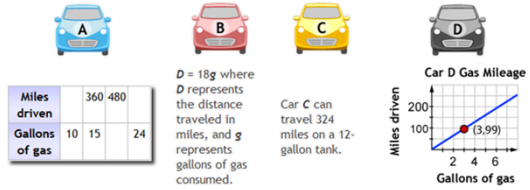
4	Distinguish between ratios that represent part:part relationships and those that demonstrate part:whole relationships. Conceive of real-world contextual situations (part:part and part:whole) to match a given ratio	• modify Engage NY resource to require students to create their own real world scenarios that illustrate the same ratio two different ways (both as a part:part relationship and also as a part:whole relationship) after the Learnzillion videos • Teaching Tip: see question #2 on exit ticket and ensure this is addressed in the lesson (interpreting part:part ratios to determine the part:whole)	1) The new floor in the school cafeteria is going to be constructed of square tiles that are either gray or white and in the pattern that appears below:  Part A: What is the ratio of gray tiles to white tiles? Answer: _____ Is this a part:part or part:whole relationship? Explain: Part B: What is the ratio of white tiles to the total number of tiles in the pattern? Answer: _____ Is this a part:part or part:whole relationship? Explain: 2) The ratio of the number of boys to the number of girls at a school is 4:5. What fraction of the students are boys? 3) Write two different real world scenarios that could be described by the ratio below: 7 to 8 Part:Part Scenario: Part:Whole Scenario:	https://learnzillion.com/lessons/580-visualize-part-to-part-ratios-using-pictures https://learnzillion.com/lessons/581-visualize-part-to-total-ratios-using-pictures https://learnzillion.com/lessons/601-identify-all-types-of-ratios-using-a-diagram Engage NY Lesson 2 (Appendix C) **Modify to meet objective*

5	Develop an intuitive understanding of equivalent ratios by using tape diagrams to explore possible quantities of each part when given the part-to-part ratio. Formalize a definition of equivalent ratios	• Students understand that a proportion represents a relationship between two quantities and may be expressed as a ratio. • Two ratios, $A:B$ and $C:D$, are equivalent ratios if there is a positive number, c, such that $C = cA$ and $D = cB$.	Create an equivalent ratio to the one listed below by drawing a tape diagram. In words, justify that your answer is correct. 3:4	Engage NY Lessons 3-4 (Appendix C) https://learnzillion.com/lessons/606-create-equivalent-ratios
6	Make the connection between the constant, c, in the definition of equivalent ratios and the value of the unit in the tape diagram used to solve ratio problems. Use tape diagrams to solve problems when given a ratio between two quantities and a change to those quantities that changes the ratio.	•	Ms. Cooper and Mr. Johnson were folding report cards to send home to parents. The ratio of the number of report cards Mr. Johnson folded to the number of report cards Ms. Cooper folded is **3:4**. At the end of the day, Mr. Johnson and Ms. Cooper folded a total of **630** report cards. How many did each person fold? Manuel said that Mr. Johnson had folded **473** report cards and Ms. Cooper folded **157** report cards because he knew Mr. Johnson had folded more report cards and folded ¾ of the total amount. Is Manuel's answer correct? Explain:	Engage NY Lessons 5 - 6 (Appendix C)

7	Use the value of a ratio to solve ratio problems in a real-world context. Determine whether or not two ratios are equivalent		Jim and Jesse each had the same amount of money. Jim spent $58 to fill the car up with gas for a road trip. Jesse spent $37 buying snacks for the trip. Afterward, the ratio of Jim's money to Jesse's money is 1:4. How much money did each have at first? Show or explain how you got your answer.	Engage NY Lesson 8 (Appendix C)
8	Represent and solve real world problems using ratio tables		Cindy was making a cake and wanted to die the batter purple. She needed to use the red and blue color die in a ratio of 3 red drops : 5 blue drops. She needs to have a total of 48 drops. Complete the ratio table below to determine how many drops of each color she needs.	Engage NY Lessons 9-10 (Appendix C)
9	**Flex Day (Instruction Based on Data)** **Recommended Resources:** "Voting for Three" (Appendix C) "Mixing Paint" (Appendix C) Engage NY Lesson 12 (Appendix C)			
10 **11**	Associate with each ratio A:B the ordered pair (A, B) and plot it in the x-y coordinate plane.	• Be sure to assess prerequisite skills of plotting on a coordinate plane before beginning this lesson (we've allotted two instructional days followed by one flex day so that you can tailor these lessons based on your students' prerequisite skills)	Create a table showing "For every 4 apples there are 5 grapes," showing up to 5 equivalent ratios. Then, plot the information on a coordinate plane.	My Math Chapter 1 Lesson 5 Engage NY Lesson 15 (Appendix C)
12	**Flex Day (Instruction Based on Data)** **Recommended Resources:** "Paper Clip Comparisons" (Appendix C) My Math Chapter 1 Problem Solving Investigation (Pages 55 – 57)			

13	Relate rates to ratios. Identify the unit rate and the rate unit.		1) What is a rate? What is a unit rate? Explain and provide examples of each	https://learnzillion.com/lessons/839 My Math Chapter 1 Inquiry Lab (Pages 27-30) Engage NY Lesson 16 (Appendix C)
14	Represent relationships between quantities in ratios, rates and unit rates. Compute unit rates.	Work with your students to identify the unit rate of quantities using a tape diagram to check against a calculation using a standard algorithm. When using large numbers, have students create these diagrams using a series of tens to represent a large amount. Check out this LearnZillion video for additional ideas.	It took Christian 27 minutes to run 3 miles. How long did it take him to run 1 mile?	My Math Chapter 1 Lesson 3 Engage NY Lesson 17 (Appendix C) https://learnzillion.com/lessons/840-create-unit-rate-using-diagram
15	Make use of the structure of division and ratios to model and interpret a rate as the division of two quantities (i.e. as a fraction)		J'Shawn works for a publishing firm. He is considered an average typist and can type 49 words/minute. If he continues at this rate, how many minutes would it take J'Shawn to type 141 words?	Engage NY Lesson 18 (Appendix C)

16	Apply unit rates to solve real world problems presented in ratio tables.		Briana helped her mother in the garden and picked 12 tomatoes in 15 minutes. The next day, she was able to pick 24 tomatoes in 30 minutes. On day 3, Briana picked 36 tomatoes. How many minutes did it take her, if she picked at the same rate as the first two days? Construct a table using the above information.	My Math Chapter 1 Lesson 4
17	Students solve unit rate problems using multiplicative, additive, and proportional reasoning.	A car magazine is writing a story about four cars. For each car, the magazine will report the number of miles driven for different amounts of gas. This table shows the number of miles driven by Car A for different amounts of gas. Car A uses gas at a constant rate. Fill in the blanks to complete the table.	(a) Ben's Game World is having a sale on video games. The store is offering a sale pack of 4 video games for $43.80. What is the unit price of a video game in the sale pack? (b) Roberto's Electronics is also having a sale on video games. The unit price of any video game in Roberto's Electronics is the same as the unit price of a video game in the sale pack at Ben's Game World. How much would it cost a customer to purchase 7 video games at Roberto's Electronics?	Engage NY Lessons 19 - 20 (Appendix C)
18	Use rates between measurements to convert measurement in one unit to measurement in another unit.		Madison rode the metro 30 miles in 180 minutes. How many hours did she ride the metro?	Engage NY Lesson 21 (Appendix C)
19	Decontextualize a given speed situation, representing symbolically the quantities involved with the formula *rate* x *time*.		Franny took a road trip to her grandmother's house. She drove at a constant speed of 60 miles per hour for 2 hours. She took a break and then finished the rest of her trip driving at a constant speed of 50 miles per hour for 2 hours. What was the total distance of Franny's trip?	Engage NY Lesson 22 (Appendix C)

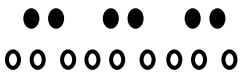
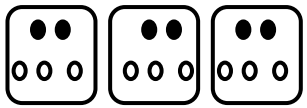
20	Solve constant rate work problems by calculating and comparing unit rates.		Lin rode a bike 20 miles in 150 minutes. If she rode at a constant speed, a. How far did she ride in 15 minutes? b. How long did it take her to ride 6 miles? c. How fast did she ride in miles per hour? d. What was her pace in minutes per mile?	Engage NY Lesson 23 (Appendix C)
21	Students make sense of and persevere in solving real world problems involving rates and ratios. Represent quantitative relationships using equivalent ratios, tape diagrams, double number line diagrams, and ratio tables.		1) A car magazine is writing a story about four cars. For each car, the magazine will report the number of miles driven for different amounts of gas. The magazine received gas mileage information for cars from several companies. Use this information to solve the problem below.  Krystal bought one of these cars. She drove 924 miles and used 28 gallons of gas. Based on her gas consumption, which car did she most likely buy? Explain your reasoning:	My Math Chapter 1 Lesson 7 Resource for Remediation: Inquiry Lab (67 – 70)

22	**Flex Day (Instruction Based on Data)** **Recommended Resources:** "Ratios and Rates" (Appendix C) "Sharing Costs: Travelling to School" (Appendix C) "Price Per Pound and Pounds Per Dollar" (Appendix C) "First Rate" (Appendix C) "Movin N' Groovin" (Appendix C) My Math Chapter 1 Review (Pages 81 – 84)			
23 **24**	Describe percent as a type of ratio that compares two different units where one of the units is 100. Model percents and write a percent as a fraction over 100 or a decimal to the hundredths place.	• understand that percents are related to part-to-whole ratios and rates where the whole is 100.	A. Write the follow ratios as a percent: 28:100 _____ 25:50 _____ B. Write the following percents as fractions: 35% _____ 77% _____ C. Write the following percents as decimals: 68% _____ 12% _____	http://learnzillion.com/lessons/593 My Math Chapter 2 Inquiry Lab (Pages 97 – 100) My Math Chapter 2 Lesson 2 Engage NY Lesson 24 (Appendix C)
25	Convert between fractions, decimals and percents.	• Since we defined percent as "per 100", encourage students to apply proportional reasoning to explain how percents can actually be greater than 100% and less than 1%	Fresh water from the lakes in the Midwest account for only 0.0003 of America's water supply. Write this number as a percent. Explain your answer using words.	My Math Chapter 2 Lessons 3-4 *Combine resources Engage NY Lesson 25 (Appendix C)

26	Compare values that are written in fraction, decimal and percent form	• Key point: in order to compare these values, we must always be referring to the same whole	Hector spends 3/8 of his time on math homework. John spends 48% of this time on math homework. Olivia spends 0.41 of her time on math homework. Who spend the most time on math? Explain and justify your answer using words, a diagram or a visual.	My Math Chapter 2 Lesson 5
27	Students find a percent of a quantity as a rate per 100.		1) Dwayne answered 80% of the questions on a quiz correctly. If he answered 40 questions correctly, what was the total number of questions on Dwayne's quiz? 2) The lot that Dana is buying for her new one-story house is 35 yards by 50 yards. Dana's house plans show that her house will cover 1,600 square feet of land. What percent of Dana's lot will **not** be covered by her house? Explain your reasoning.	My Math Chapter 2 Lesson 7 "Free Throws" "Reaching the Goal" (Appendix C)
28	Given a part and a percent, create an equation and/or construct a model to solve for the unknown whole.		1a) Sam purchased 3 games for $140 after a discount of 30%. What was the original price? 1b) If Sam had used a 20% off coupon and opened a frequent shopped discount membership to save 10%, would the games still have a total of $140? 2) Micah read 16 pages of his book. If this is 10% of the book, how many pages are in the book?	Engage NY Lessons 26-27 (Appendix C) **Modify resource to require students to set up equations* https://learnzillion.com/lessons/597-find-the-total-when-the-percent-and-part-are-known

29	Make sense of and persevere in solving real world problems involving percents.	• Key point: Percent problems include the part, whole, and percent. When one of these values is missing, we can use tables, diagrams, equations and models to solve for the missing number.	1) The Sparkling House Cleaning Company has cleaned 28 houses this week. If this number represents 40% of the total number of houses they are contracted to clean, how many total houses will the company clean by the end of the week?	My Math Chapter 2 Lesson 8 http://learnzillion.com/lessons/598x
30	Make sense of and persevere in solving multi-step, real world problems involving percents.	• Today's problems will include sales tax, interest, discounts, etc. so that students have an opportunity to practice multi-step problems involving percentages they are most likely to encounter as consumers	1) Selina bought a shirt on sale that was 20% less than its original price. The original price was $5 more than the sale price. What was the original price? 2) In the first week, 5,000 people visited a book fair. The number of visitors increased by 10% in the second week. How many people visited the book fair in the second week?	"Anna in DC Task" (Appendix C) "Ice Cream or Cake?" (Appendix C)
31	**Flex Day (Instruction Based on Data)** **Recommended Resources:** "Bake Sale Brownies" (Appendix C) "The Rocky Mountain Vacation Trip" (Appendix C) My Math Chapter 2 Review (Pages 165 -168) My Math Unit 1 Project (Pages 169 – 170)			
32	**MCLASS Beacon End of Unit Assessment** **Appendix B** ***This assessment will be administered online**			

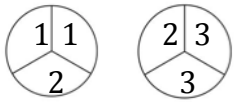
Appendix A:
Unpacked Standards Guide
Source: Public Schools of North Carolina NCDPI Collaborative Workspace

Standard	**Unpacking** *What do these standards mean a child will know and be able to do?*
6.RP.1 Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. *For example, "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes."*	6.RP.1 A ratio is the comparison of two quantities or measures. The comparison can be part-to-whole (ratio of guppies to all fish in an aquarium) or part-to-part (ratio of guppies to goldfish). Example 1: A comparison of 6 guppies and 9 goldfish could be expressed in any of the following forms: $\frac{6}{9}$, 6 to 9 *or* 6:9. If the number of guppies is represented by black circles and the number of goldfish is represented by white circles, this ratio could be modeled as  These values can be regrouped into 2 black circles (guppies) to 3 white circles (goldfish), which would reduce the ratio to, $\frac{2}{3}$, 2 to 3 or 2:3.  Students should be able to identify and describe any ratio using "For every _____ ,there are _____" In the example above, the ratio could be expressed saying, "For every 2 guppies, there are 3 goldfish". NOTE: Ratios are often expressed in fraction notation, although ratios and fractions do not have identical meaning. For example, ratios are often used to make "part-part" comparisons but fractions are not.
6.RP.2 Understand the concept of a unit rate a/b associated with a ratio a:b with b ≠ 0, and use rate language in the context of a ratio relationship. *For example, "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is ¾ cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger."*[1]	6.RP.2 A unit rate expresses a ratio as part-to-one, comparing a quantity in terms of one unit of another quantity. Common unit rates are cost per item or distance per time. Students are able to name the amount of either quantity in terms of the other quantity. Students will begin to notice that related unit rates (i.e. miles / hour and hours / mile) are reciprocals as in the second example below. At this level, students should use reasoning to find these unit rates instead of an algorithm or rule. In 6th grade, students are not expected to work with unit rates expressed as complex fractions. Both the numerator and denominator of the original ratio will be whole numbers.

[1] Expectations for unit rates in this grade are limited to non-complex fractions.

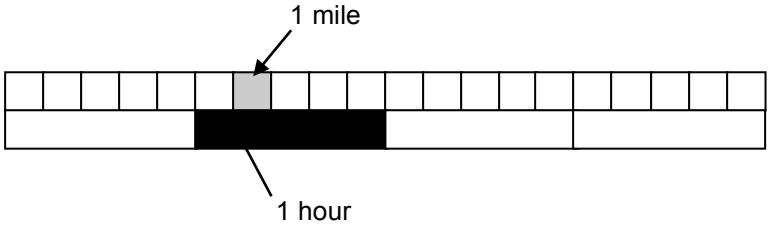
Example 1:
There are 2 cookies for 3 students. What is the amount of cookie each student would receive? (i.e. the unit rate)

Solution: This can be modeled as shown below to show that there is $\frac{2}{3}$ of a cookie for 1 student, so the unit rate is $\frac{2}{3}$: 1.



Example 2:
On a bicycle Jack can travel 20 miles in 4 hours. What are the unit rates in this situation, (the distance Jack can travel in 1 hour and the amount of time required to travel 1 mile)?

Solution: Jack can travel 5 miles in 1 hour written as $\frac{5\,mi}{1\,hr}$ and it takes $\frac{1}{5}$ of a hour to travel each mile written as $\frac{\frac{1}{5}\text{hr}}{1\text{mi}}$. Students can represent the relationship between 20 miles and 4 hours.



6.RP.3 Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations.

a. Make tables of equivalent ratios relating quantities with whole- number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios.

6.RP.3 Ratios and rates can be used in ratio tables and graphs to solve problems. Previously, students have used additive reasoning in tables to solve problems. To begin the shift to proportional reasoning, students need to begin using multiplicative reasoning. Scaling up or down with multiplication maintains the equivalence. To aid in the development of proportional reasoning the cross-product algorithm is *not* expected at this level. When working with ratio tables and graphs, *whole number* measurements are the expectation for this standard.

Example 1:
At Books Unlimited, 3 paperback books cost $18. What would 7 books cost? How many books could be purchased with $54.

Solution: To find the price of 1 book, divide $18 by 3. One book costs $6. To find the price of 7 books, multiply $6 (the cost of one book times 7 to get $42. To find the number of books that can be purchased with $54, multiply $6 times 9 to get $54 and then multiply 1 book times 9 to get 9 books. Students use ratios, unit rates and multiplicative reasoning to solve problems in various contexts, including measurement, prices, and geometry. Notice in the table below, a multiplicative relationship exists between the numbers both horizontally (times 6) and vertically (ie. 1 • 7 = 7; 6 • 7 = 42). Red numbers indicate solutions.

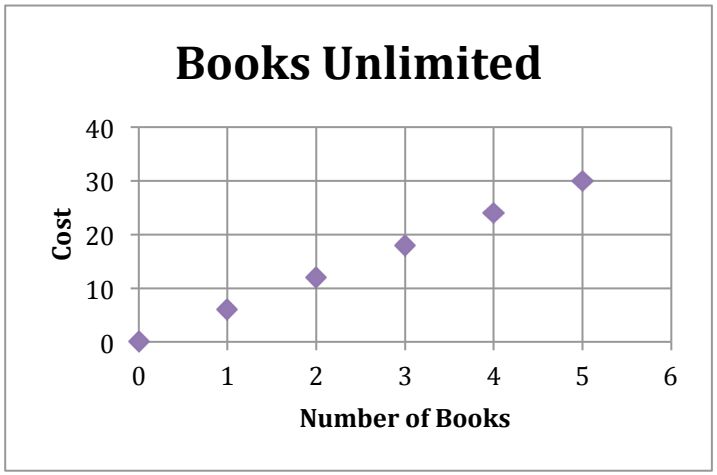
Number of Books (n)	Cost (C)

1	6
3	18
7	42
9	54

Students use tables to compare ratios. Another bookstore offers paperback books at the prices below. Which bookstore has the best buy? Explain your answer.

Number of Books (n)	Cost (C)
4	20
8	40

To help understand the multiplicative relationship between the number of books and cost, students write equations to express the cost of any number of books. Writing equations is foundational for work in 7th grade. For example, the equation for the first table would be $C = 6n$, while the equation for the second bookstore is $C = 5n$.
The numbers in the table can be expressed as ordered pairs (number of books, cost) and plotted on a coordinate plane.
Students are able to plot ratios as ordered pairs. For example, a graph of Books Unlimited would be:



Example 2:

Ratios can also be used in problem solving by thinking about the total amount for each ratio unit.
The ratio of cups of orange juice concentrate to cups of water in punch is 1: 3. If James made 32 cups of punch, how many cups of orange did he need?

Solution: Students recognize that the total ratio would produce 4 cups of punch. To get 32 cups, the ratio would need to be duplicated 8 times, resulting in 8 cups of orange juice concentrate.

Example 3:
Using the information in the table, find the number of yards in 24 feet.

Feet	3	6	9	15	24
Yards	1	2	3	5	?

Solution:
There are several strategies that students could use to determine the solution to this problem:

- Add quantities from the table to total 24 feet (9 feet and 15 feet); therefore the number of yards in 24 feet must be 8 yards (3 yards and 5 yards).
- Use multiplication to find 24 feet: 1) 3 feet x 8 = 24 feet; therefore 1 yard x 8 = 8 yards, or 2) 6 feet x 4 = 24 feet; therefore 2 yards x 4 = 8 yards.

Example 4:
Compare the number of black circles to white circles. If the ratio remains the same, how many black circles will there be if there are 60 white circles?

Black	4	40	20	60	?
White	3	30	15	45	60

Solution:
There are several strategies that students could use to determine the solution to this problem

- Add quantities from the table to total 60 white circles (15 + 45). Use the corresponding numbers to determine the number of black circles (20 + 60) to get 80 black circles.
- Use multiplication to find 60 white circles (one possibility 30 x 2). Use the corresponding numbers and operations to determine the number of black circles (40 x 2) to get 80 black circles.

b. Solve unit rate problems including those involving unit pricing and constant speed. *For example, if it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?*

Students recognize the use of ratios, unit rate and multiplication in solving problems, which could allow for the use of fractions and decimals.

Example 1:
In trail mix, the ratio of cups of peanuts to cups of chocolate candies is 3 to 2. How many cups of chocolate candies would be needed for 9 cups of peanuts?

Peanuts	Chocolate
3	2

Solution:
One possible solution is for students to find the number of cups of chocolate candies for 1 cup of peanuts by dividing both sides of the table by 3, giving $\frac{2}{3}$ cup of chocolate for each cup of peanuts. To find the amount of chocolate needed for 9 cups of peanuts, students multiply the unit rate by nine ($9 \bullet \frac{2}{3}$), giving 6 cups of chocolate.

Example 2:
If steak costs $2.25 per pound, how much does 0.8 pounds of steak cost? Explain how you determined your answer.

Solution:
The unit rate is $2.25 per pound so multiply $2.25 x 0.8 to get $1.80 per 0.8 lb of steak.

c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent.

This is the students' first introduction to percents. Percentages are a rate per 100. Models, such as percent bars or 10 x 10 grids should be used to model percents.

Students use ratios to identify percents.

Example 1:
What percent is 12 out of 25?
Solution: One possible solution method is to set up a ratio table:
Multiply 25 by 4 to get 100. Multiplying 12 by 4 will give 48, meaning that 12 out of 25 is equivalent to 48 out of 100 or 48%.

Part	Whole
12	25
?	100

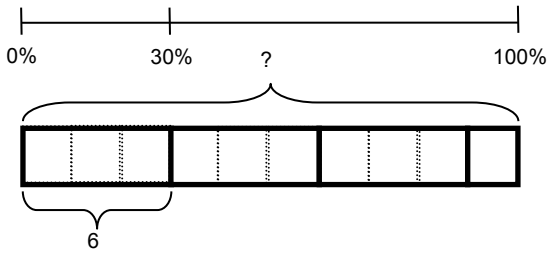
Students use percentages to find the part when given the percent, by recognizing that the whole is being divided into 100 parts and then taking a part of them (the percent).

Example 2:
What is 40% of 30?

Solution: There are several methods to solve this problem. One possible solution using rates is to use a 10 x 10 grid to represent the whole amount (or 30). If the 30 is divided into 100 parts, the rate for one block is 0.3. Forty percent would be 40 of the blocks, or 40 x 0.3, which equals 12.
See the web link below for more information.
http://illuminations.nctm.org/LessonDetail.aspx?id=L249

Students also determine the whole amount, given a part and the percent.

Example 3:
If 30% of the students in Mrs. Rutherford's class like chocolate ice cream, then how many students are in Mrs. Rutherford's class if 6 like chocolate ice cream?



(*Solution:* 20)

Example 4:
A credit card company charges 17% interest fee on any charges not paid at the end of the month. Make a ratio table to show how much the interest would be for several amounts. If the bill totals $450 for this month, how much interest would you have to be paid on the balance?
Solution:

Charges	$1	$50	$100	$200	$450
Interest	$0.17	$8.50	$17	$34	?

One possible solution is to multiply 1 by 450 to get 450 and then multiply 0.17 by 450 to get $76.50.

d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

A ratio can be used to compare measures of two different types, such as inches per foot, milliliters per liter and centimeters per inch. Students recognize that a conversion factor is a fraction equal to 1 since the numerator and denominator describe the same quantity. For example, $\frac{12 \text{ inches}}{1 \text{ foot}}$ is a conversion factor since the numerator and denominator equal the same amount. Since the ratio is equivalent to 1, the identity property of multiplication allows an amount to be multiplied by the ratio. Also, the value of the ratio can also be expressed as $\frac{1 \text{ foot}}{12 \text{ inches}}$ allowing for the conversion ratios to be expressed in a format so that units will "cancel".

Students use ratios as conversion factors and the identity property for multiplication to convert ratio units.

	Example 1: How many centimeters are in 7 feet, given that 1inch ≈ 2.54 cm. *Solution:* $7 \text{ feet} \times \frac{12 \text{ inches}}{1 \text{ foot}} \times \frac{2.54 \text{ cm}}{1 \text{ inch}} = 7 \cancel{\text{feet}} \times \frac{12 \cancel{\text{inches}}}{1 \cancel{\text{foot}}} \times \frac{2.54 \text{ cm}}{1 \cancel{\text{inch}}} = 7 \times 12 \times 2.54 \text{ cm} = 213.36 \text{ cm}$ Note: Conversion factors will be given. Conversions can occur both between and across the metric and English systems. Estimates are not expected.